Malaysian Journal of Mathematical Sciences 18(3): 647–661 (2024)
https://doi.org/10.47836/mjms.18.3.12

Malaysian Journal of Mathematical Sciences

Journal homepage: https://mjms.upm.edu.my

Triangular Fuzzy Cognitive Maps Method for Modelling Interrelation between Causal and Trigger Factors Towards Students' Mathematics Problem-Solving Ability

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Received: 18 January 2024
Accepted: 10 June 2024

Abstract

The purpose of this study is to model the interrelation between causal and trigger factors towards students' mathematics problem-solving ability by using the triangular fuzzy cognitive maps (TrFCM) method. Selection weaknesses and limitations in the method of relational analysis cause the interrelation and influence between variables not to be visualised and do not reveal the characteristics of the actual interaction. As a result, this study demonstrates TrFCM as a more effective way of analysing the relationship between variables based on the complexity that happens in analysing causal factors and triggers for students' problem-solving abilities in mathematics. The results of the influential relations map (IRM) demonstrate that emotion and metacognition are the triggers for problem-solving ability. While executive function is the main cause of success in completing mathematics problems, it is also influenced by additional factors such as motivation, attention, and working memory. These causal and triggering factors also mobilise parts of students' cognitive and behavioural performance to improve the process of solving mathematics problems. Based on the outcomes of this study, computational intelligence methods like fuzzy systems give useful procedures for analysing data from expert surveys. The TrFCM method offers a more accurate relational analysis procedure in modelling interrelation between human factors.

Keywords: triangular fuzzy cognitive maps; students' ability; causal and trigger factors; relational analysis; influential relations map.

1 Introduction

Relational analysis is a computational technique that is required to describe a phenomenon, especially in explaining or modelling cause or effect. Relational analysis allows more interpretations to be made compared to conceptual analysis [29]. This is because this analysis can go beyond the frequency of individual concepts and can also be interpreted in the form of inference that explains the meaning as a whole. Among the phenomena that can be explained using relational analysis are those related to human factors such as the modelling of the learning process. Cognitive maps are a subcategory in relational analysis that is often used. According to Palmquist and Sokoll [30], cognitive maps are qualitative models of a system, which is one of the relational analysis methods that use visual representation across text. Cognitive maps are a very efficient platform for modelling complex relationships between variables, where the mapping results are flexible and can be examined, compared and discussed [29].

The development of computational intelligence such as fuzzy systems leads to improvements to the cognitive maps system. Triangular fuzzy cognitive maps (TrFCM) were introduced to overcome the limitations of the existing cognitive maps system. Selvam [37] traces the development of TrFCM from Lotfi A. Zadeh's introduction of fuzzy cognitive maps (FCM), which Axelord later applied in political and social science decision-making, followed by Kosko's integration of fuzzy value and fuzzy degrees into the system. The advantage of TrFCM is that it can work and still be able to produce reasonable results from the limited number of experts involved in making the assessment [9]. In addition, input from a large and diverse group of experts can also be easily integrated despite the limitations of expert opinion and group thinking at different levels [23].

The learning process is very complex in describing the relationship between causal or trigger factors as well as the interaction between consequences and effects. Controversy began to arise when the analysis of this relationship only displayed numerical analysis that did not describe the interrelation more meaningfully [33, 34]. This limitation is due to the selection of inappropriate methods of analysing the relationship between factors and the displayed results do not achieve the objective of the analysis [3, 33, 34]. In addition, the existing knowledge and practical gaps add to this confusion among educators, especially in applying intelligent analysis methods such as fuzzy analytic and multi-criteria decision-making methods [4, 11]. Therefore, the analysis of this interaction is very much in need of relational analysis methods such as FCM.

In the context of the learning process, methods of analysing and modelling factors related to students' mathematics problem-solving ability (SMPSA) still receive criticism [34]. According to some researchers, SMPSA involves several causal and trigger factors such as cognition, behavioural performance [8, 14, 21], motivation [25], emotion [8, 14, 25], attention [4, 38], metacognition, working memory, and executive function [8, 25]. These factors are difficult to interpret, especially to see how the process or mechanism occurs before and during students solve mathematics problems [4, 16]. For this reason, in describing the real thing and modelling the factors, more effective methods are needed such as intelligent analysis methods, namely relational analysis such as TrFCMs.

One more thing is that knowledge analysis involving individuals with different levels of thinking about SMPSA is also a source of limitations in the problem of modelling causal and trigger factors. This happens when the perception survey about SMPSA is carried out, it involves the views of educators as individuals who are directly involved in this field. In addition, the views of academic experts are also a priority. So here there will be a clash of two different points of view based on the perspective of practice (educators) and the perspective of theory (experts). This gap is discussed by Özesmi and Özesmi [29] and Jetter and Kok [23] who state that no way of modelling can be implemented in the absence of scientific data involving a combination of local knowledge from individuals who are very suitable in the ecosystem.

In conclusion, there is a gap such as the issue of inappropriate analytical methods, limitations of the method and the clash of opinions (cognitive) of individuals involved in this modelling process. So, there is a need for an intelligent analysis approach that can reduce the gap. TrFCM is a suitable choice, based on the ability to work with a limited number of experts [9], the ability to combine levels of thought [29] and the ability to describe input factors (concepts) through a triggering pattern-induced map [23,24]. Therefore, the main objective of this paper is to introduce the TrFCM method for modelling the interrelation between causal and trigger factors towards SMPSA. The present study offers a more accurate relational analysis procedure in modelling the interrelation between human factors. This paper's main contributions include:

1. Demonstrate how the TrFCM methods can be used to analyse data from expert surveys.
2. Introducing a more effective relational analysis procedure to extract causal and trigger factors of the SMPSA.

The proposed methodology consists of fuzzy cognitive map methods under fuzzy analytics and multi-criteria decision-making methods. First, with the guidance and content analysis from the literature review, all the causal and trigger factors are gathered. Then, FCM are used to calculate the weightage of each attribute and map the influential relations among them. The rest of the paper is organised as follows: In Section 2, the literature review summarises fuzzy triangular numbers, FCM and students' mathematics problem-solving abilities. A procedure of TrFCM is discussed in Section 3. In Section 4, an empirical case conducted is presented to demonstrate the proposed method. Finally, conclusions and future research directions are provided in Section 5.

2 Literature Review

2.1 Triangular fuzzy number

In general, the description of triangular fuzzy number, A is as follows.

Definition 2.1. [43] *A triangular fuzzy number A can be defined by a triplet (a_1, a_2, a_3). The membership function $\mu_A(x)$ is*

$$\mu_A(x) = \begin{cases} 0, & x < a_1, \\ \dfrac{x-a_1}{a_2-a_1}, & a_1 \leq x \leq a_2, \\ \dfrac{x-a_3}{a_2-a_3}, & a_2 \leq x \leq a_3, \\ 0, & a_3 < x, \end{cases}$$

where $0 \leq a_1 \leq a_2 \leq a_3 \leq 1$, the value of a_1 and a_3 respetively for the lower and upper values of A, and a_2 is the middle value.

Definition 2.2. [42] *For triangular fuzzy numbers $A = (a_1, a_2, a_3)$ and $B = (b_1, b_2, b_3)$, where $* = \{+, -, \times, \div\}$ be the arithmetic operations on the triangular fuzzy numbers are defined by $A * B = \{a_i * b_j, a_i \in A, b_j \in B\}$. In particular, for any two triangular fuzzy numbers $A = (a_1, a_2, a_3)$ and $B = (b_1, b_2, b_3)$, then* *i. Addition* $(+)$*:* $A + B = (a_1 + b_1, a_2 + b_2, a_3 + b_3)$.

ii. Subtraction $(-)$*:* $A - B = (a_1 - b_1, a_2 - b_2, a_3 - b_3)$.

iii. Multiplication $(\times)$*:* $k \times A = (ka_1, ka_2, ka_3),\ k \in \mathbb{R},\ k \geq 0,\ A \times B = (a_1b_1, a_2b_2, a_3b_3)$.

iv. Division $(\div)$*:* $A^{-1} = (a_1, a_2, a_3)^{-1} \cong \left(\frac{1}{a_3}, \frac{1}{a_2}, \frac{1}{a_1}\right),\ a_1 > 0,\ a_2 > 0,\ a_3 > 0,$

$A \div B \cong \left(\frac{a_1}{b_3}, \frac{a_2}{b_2}, \frac{a_3}{b_1}\right), a_1 \geq 0, b_1 \geq 0.$

2.2 Triangular fuzzy cognitive maps (TrFCM)

TrFCM are more suitable for implementation with unsupervised data orientation [24,35]. TrFCM can function and operate with a minimum of three expert opinions [9]. TrFCM will model every feed item as a set of attributes and will map the causal relationships among them. In addition, the implementation of this method becomes very simple and effective because it can analyse data through directed graphs and connection matrices [24]. This efficiency is very different compared to the conventional FCM model, that is, usually FCM only builds relationships and ON-OFF positions between attributes [35,37]. However, through this TrFCM, the position and intertwining of the causes of the problem can be extracted more accurately by only using the weighting of the attribute.

Remark 2.1. *When the TrFCM nodes are fuzzy sets, they are referred to as fuzzy triangular nodes.*

Remark 2.2. *Simple triangular FCMs are those that have edge weights or causalities from the set* $\{-1, 0, 1\}$.

Remark 2.3. *A TrFCM is a directed graph that consists of nodes representing concepts such as attributes and criteria, and edges representing causal relationships between these concepts.*

Remark 2.4. *When there is feedback in a TrFCM, or when the causal relations flow through a cycle in a revolutionary way, the TrFCM is called a dynamical system.*

Remark 2.5. *A fixed point is the equilibrium state of a dynamical system that is a unique state vector. Consider a TrFCM with nodes* $TrC_1, TrC_2., ..., TrC_n$.

Remark 2.6. *If the TrFCM settles down with a state vector repeating in the form* $A_1 > A_2 > A_i, ..., > A_1$*, then this equilibrium is called a limit cycle.*

FCMs gained popularity due to their ability to represent causal relationships in a fuzzy logic framework [24]. Over time, FCMs have undergone several developments and extensions.

Researchers have explored various extensions, including interval type-2 fuzzy cognitive maps, which handle uncertainty more effectively by incorporating interval type-2 fuzzy sets [2,6]. Additionally, hybridisation approaches have emerged, such as neuro-fuzzy cognitive maps, which integrate neural network principles with FCMs to enhance learning and adaptation capabilities [7,12]. These advancements have expanded the applicability and robustness of FCMs, making them valuable tools for modelling and decision-making in diverse domains such as social sciences [2,9,24], bioinformatics [5,6], robotics [7], intelligent systems controller [12,24] and etc.

2.3 Students' mathematics problem-solving ability (SMPSA)

SMPSA is an important aspect of mathematics education [36]. It is used as evidence of student performance in each country as implemented in international programs such as Trend in International Mathematics and Science Study (TIMSS) and Programme of International Student Achievement (PISA) [1]. This level of ability is an index of student success in learning mathematics and can be used as a measurement determinant based on level [17]. The higher the level of students' ability to solve problems, the better their mathematics learning performance [36,39]. According to Simamora et al. [38], the capacity to respond to problems in mathematics is highly explicit and should be stressed because it is the only mathematical ability that is deemed excellent as it is both an ability and knowledge.

The mathematics problem-solving ability as stated in the Standard Document for Curriculum and Assessment of Mathematics [26] is:

i. Formulate the problem accurately and identify the main issue of the problem.

ii. Present a solution clearly and make explicit the assumptions made.

iii. Solving difficult problems by analysing smaller, more specific problems.

iv. Be open-minded and use different approaches to solve the same problem.

v. Solving problems with confidence even when the solution is not obvious.

vi. Ask for help if needed.

Based on this information, the ability to solve mathematics problems is a mechanism that involves various factors such as cognition and behaviour [10,27,32]. It is a mental process and requires actions by students such as understanding the problem, describing the problem based on certain assumptions, and then solving the problem according to the right steps or approach [1]. In this context, Garcia et al. [17] explain that solving mathematics problems is a competency, which refers to the ability to manage mental processes such as reduction, revision, and exploration. It can be achieved by consciously determining relationships and designing problem-solving actions based on appropriate heuristics. The involvement of students in solving mathematics problems ensures that they can interpret objects, explore, identify and describe the characteristics of relationships, inequalities and even support [38]. Therefore, this ability involves the causative factor and also the factor that triggers students' excitement to determine and find a solution to each problem assigned to them.

The ability to solve problems is also based on a strong relationship with brain intelligence [26,39]. However, conceptually this matter is still not clear, especially concerning the form of interaction between the factors involved [2,31,32]. This needs to be visualised by detailing what cycle students experience and do before, during and after they are assigned a problem situation or mathematics question. According to Schoenfeld [36], solving mathematics problems is a systematic and planned action, so it is clear that it is related to certain causal factors and triggers that will affect the work of students. Previous studies have broken down the causal and trigger factors for the ability to solve mathematics problems which are cognition, behavioural performance [8,14,28], motivation [25,28], emotion [8,14,25], attention [4,38], metacognition, working memory, and executive function [8,25].

More deeply, it is a chain of processes and relationships that are influenced by beliefs, desires, wills, and motivations. This influential relationship is related to the knowledge processing mechanism involving emotion, attention, cognitive, metacognitive, executive function, and working memory processes and then ends with follow-up behavioural actions as a complement. This hypothesis has been discussed respectively by Schoenfeld [36] and Anderson [10] in their theory where it has been stated that changes in the learning process in individuals begin with the formation of belief systems, and desires and are followed by cognitive and behavioural actions based on the form of the situation or revealed assignments.

3 Methodology

The main purpose of this study is to model an interrelation between causal and trigger factors towards students' mathematics problem-solving ability. This model will be illustrated through an influential relations map (IRM) and their influence weights to explain the strength of the relationship. In the first stage, content analysis was carried out to see and determine what are the causal and trigger factors for students' mathematics problem-solving ability through a literature review. Next in the second stage, a semi-quantitative interview involving 5 experts using unstructured questions was administered to obtain expert consensus and determine the strength of the relationship between causal and trigger factors for SMPSA. Figure 1 illustrates the procedures and steps implemented.

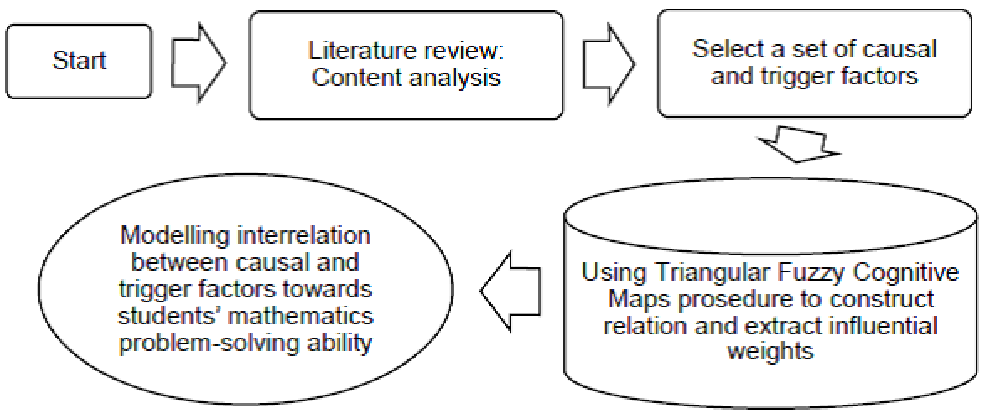

Figure 1: Flowchart of the proposed procedure of the TrFCM model.

The steps in implementing TrFCM are as follows:

Step 1: Prepare a $n \times n$ fuzzy matrix, which is called the connection matrix by using linguistic values, as listed in Table 1.

Table 1: The linguistic values.

Linguistic values	Weightage of TrFCM	Average of TrFCM
Very low (VL)	(0.0, 0.0, 0.25)	0.08
Low (L)	(0.0, 0.25, 0.5)	0.25
Medium (M)	(0.25, 0.5, 0.75)	0.50
High (H)	(0.5, 0.75, 1.0)	0.75
Very high (VH)	(0.75, 1.0, 1.0)	0.92

Step 2: Prepare the maximum weightage of the matrix using $Tr(M)$.

Step 3: Find the limit cycle.

Let $TrC_1, TrC_2, \dots, TrC_n$ be the nodes of a TrFCM. Here $Tr(M)$ be an adjacency matrix. Consider the instantaneous state vector as $A_1 = (1, 0, 0, \dots, 0)$ for $A_1Tr(M)$ is switched ON. $A_1Tr(M) = a_1, a_2, \dots, a_n$ will get a triangular vector.

Step 4: The threshold operation is denoted by $\rightarrow A_1Tr(M)max(weight)$. That is by replacing a_i by 1 if a_i is the maximum weight of the triangular node (ie., $a_i = 1$), otherwise a_i by 0 (ie., $a_i = 0$).

Step 5: Suppose $A_1Tr(M)max(weight) = A_2$. Then, consider $A_2Tr(M)$ weight is the ON attribute triangular vector. Find $A_2Tr(M)$.

Step 6: Find $A_2Tr(M)_{sum}$ (ie., summing of the expert opinion of each attribute).

Step 7: The threshold operation is denoted by $\rightarrow A_2Tr(M)max(weight)$. That is by replacing a_i by 1 if a_i is the maximum weight of the triangular node (ie., $a_i = 1$), otherwise a_i by 0 (ie., $a_i = 0$). If the $A_1Tr(M)max(weight) = A_2Tr(M)max(weight)$. Then, the dynamical system ends, otherwise repeat the same procedure.

Step 8: This procedure is repeated till we get a limit cycle or a fixed point.

4 Results and Discussion

The following are the steps in the study, results and discussion of findings.

Step 1: The first stage is to explore problems in SMPSA through a literature review and the following Table 2, are the results of the content analysis obtained.

Table 2: Analysis of causal and trigger factors (attributes).

Nodes of TrFCM, TrC_i	Attributes	Description of attributes	References
C_1	Cognition	Acquiring knowledge and comprehension through cognition, experience, and the senses is a mental action or process. It includes all aspects of cognitive functions and processes such as perception, attention, imagination, knowledge and memory creation, judgement, reasoning, calculation, problem-solving and decision-making, and language understanding and production.	[8,14,21,22,28]
C_2	Behavioural performance	How one acts or conducts oneself. The actions that are believed to support the ability to assess each of the key performance questions.	[8,14,18,28]
C_3	Emotion	This term refers to either positive or negative self-talk. Affects the student's attention, motivation to study, choice of learning strategies, self-regulation of learning, and academic achievement.	[8,14,19,25]
C_4	Motivation	Refer to individuals' ideas of autonomy and the motives they have for acting in a certain situation. A feeling of willingness, need, want, and compulsion.	[15,20,25,28]
C_5	Attention	Refers to human biological systems and complicated cognitive functions that tend to focus on distinguishing features when processing massive amounts of information. Also known as the belief system of humans.	[4,13,38]
C_6	Executive function	The ability to manipulate objects intellectually, to evaluate, prepare, and strategize. Key components to organisational success, decision-making, and life choices. Other memory systems are provided with cognitive resources.	[14,20,22,25,41]
C_7	Metacognition	Refers to the ability to plan, create goals, and allocate resources before learning, as well as the ability to monitor and reflect on what new things are learnt.	[14,22,25,41]
C_8	Working memory	The ability to retain information and recall it later, to process incoming information accurately and rapidly, and to appraise one's ability to understand something. Include attention control.	[15,25,40,41]

Step 2: In this step, prepare a fuzzy matrix called the connection matrix by using linguistic variables related to the fuzzy cognitive map.

$$Tr(M) = \begin{vmatrix} 0 & H & M & H & H & VH & H & H \\ H & 0 & M & M & VH & H & H & M \\ H & VH & 0 & VH & M & M & M & M \\ M & H & VH & 0 & M & H & M & M \\ H & H & H & M & 0 & H & VH & H \\ H & H & M & M & M & 0 & H & VH \\ VH & H & M & M & VH & H & 0 & H \\ H & H & M & M & M & VH & H & 0 \end{vmatrix}.$$

Step 3: Prepare the maximum weightage of the matrix using the linguistic listed in Table 1. The results shown in Table 3 and then Table 4 are the average weighting matrix formed.

Table 3: Connection matrix with linguistic value.

$Tr(M)$	$Tr(C_1)$	$Tr(C_2)$	$Tr(C_3)$	$Tr(C_4)$	$Tr(C_5)$	$Tr(C_6)$	$Tr(C_7)$	$Tr(C_8)$
$Tr(C_1)$	0	(0.5,0.75,1)	(0.25,0.5,0.75)	(0.5,0.75,1)	(0.5,0.75,1)	(0.75,1,1)	(0.5,0.75,1)	(0.5,0.75,1)
$Tr(C_2)$	(0.5,0.75,1)	0	(0.25,0.5,0.75)	(0.25,0.5,0.75)	(0.75,1,1)	(0.5,0.75,1)	(0.5,0.75,1)	(0.25,0.5,0.75)
$Tr(C_3)$	(0.5,0.75,1)	(0.75,1,1)	0	(0.75,1,1)	(0.25,0.5,0.75)	(0.25,0.5,0.75)	(0.25,0.5,0.75)	(0.25,0.5,0.75)
$Tr(C_4)$	(0.25,0.5,0.75)	(0.5,0.75,1)	(0.75,1,1)	0	(0.25,0.5,0.75)	(0.5,0.75,1)	(0.25,0.5,0.75)	(0.25,0.5,0.75)
$Tr(C_5)$	(0.5,0.75,1)	(0.5,0.75,1)	(0.5,0.75,1)	(0.25,0.5,0.75)	0	(0.5,0.75,1)	(0.75,1,1)	(0.5,0.75,1)
$Tr(C_6)$	(0.5,0.75,1)	(0.5,0.75,1)	(0.25,0.5,0.75)	(0.25,0.5,0.75)	(0.25,0.5,0.75)	0	(0.5,0.75,1)	(0.75,1,1)
$Tr(C_7)$	(0.75,1,1)	(0.5,0.75,1)	(0.25,0.5,0.75)	(0.25,0.5,0.75)	(0.75,1,1)	(0.5,0.75,1)	0	(0.5,0.75,1)
$Tr(C_8)$	(0.5,0.75,1)	(0.5,0.75,1)	(0.25,0.5,0.75)	(0.25,0.5,0.75)	(0.25,0.5,0.75)	(0.75,1,1)	(0.5,0.75,1)	0

Table 4: Average weightage of $Tr(M)$.

$Tr(M)$	$Tr(C_1)$	$Tr(C_2)$	$Tr(C_3)$	$Tr(C_4)$	$Tr(C_5)$	$Tr(C_6)$	$Tr(C_7)$	$Tr(C_8)$
$Tr(C_1)$	0	0.75	0.5	0.75	0.75	0.92	0.75	0.75
$Tr(C_2)$	0.75	0	0.5	0.5	0.92	0.75	0.75	0.5
$Tr(C_3)$	0.75	0.92	0	0.92	0.5	0.5	0.5	0.5
$Tr(C_4)$	0.5	0.75	0.92	0	0.5	0.75	0.5	0.5
$Tr(C_5)$	0.75	0.75	0.75	0.5	0	0.75	0.92	0.75
$Tr(C_6)$	0.75	0.75	0.5	0.5	0.5	0	0.75	0.92
$Tr(C_7)$	0.92	0.75	0.5	0.5	0.92	0.75	0	0.75
$Tr(C_8)$	0.75	0.75	0.5	0.5	0.5	0.92	0.75	0

Step 4: Find the limit cycle. Let $Tr(C_1)$ be in the ON state and other nodes in the OFF state.

$$
\begin{aligned}
A^{(1)} &= (1,0,0,0,0,0,0,0),\\
A^{(1)}Tr(C_1)weight &= \{(0),(0.5,0.75,1),(0.25,0.5,0.75),(0.5,0.75,1),\\
&\quad (0.5,0.75,1),(0.75,1,1),(0.5,0.75,1),(0.5,0.75,1)\},\\
A^{(1)}Tr(C_1)average &= \{(0),(0.75),(0.5),(0.75),(0.75),(0.92),(0.75),(0.75)\},\\
\rightarrow A^{(1)}Tr(C_1)max(weight) &= (0,0,0,0,0,1,0,0) = A_1^{(1)},\\
A_1^{(1)}Tr(C_1)average &= \{(0.69),(0.69),(0.46),(0.46),(0.46),(0),(0.69),(0.8464)\},\\
\rightarrow A_1^{(1)}Tr(C_1)max(weight) &= (0,0,0,0,0,0,0,1) = A_2^{(1)},\\
A_2^{(1)}Tr(C_1)average &= \{(0.6348),(0.6348),(0.4232),(0.4232),(0.4232),(0.7787),\\
&\quad (0.6348),(0)\},\\
\rightarrow A_2^{(1)}Tr(C_1)max(weight) &= (0,0,0,0,0,1,0,0) = A_3^{(1)} = A_1^{(1)}.
\end{aligned}
$$

Step 5: Repeat the procedure for the other nodes.

Table 5: Total weightage of the attributes.

	$Tr(C_1)$	$Tr(C_2)$	$Tr(C_3)$	$Tr(C_4)$	$Tr(C_5)$	$Tr(C_6)$	$Tr(C_7)$	$Tr(C_8)$	Triggering pattern
(1,0,0,0,0,0,0,0)	2.7572	2.7572	1.8381	1.8381	1.8381	3.3821	2.7572	0.0000	*C6>C8>C6*
(0,1,0,0,0,0,0,0)	0.9522	0.9522	0.6348	0.6348	0.6348	1.168	0.9522	0.0000	*C5>C7>C1,C5>C6,C7>C1, C8>C6>C8>C6*
(0,0,1,0,0,0,0,0)	0.876	0.876	0.584	0.584	0.584	1.0746	0.876	0.0000	*C2,C4>C6>C8>C6*
(0,0,0,1,0,0,0,0)	2.9969	2.9969	1.9979	1.9979	1.9979	3.6762	2.9969	0.0000	*C3>C2,C4>C6>C8>C6*
(0,0,0,0,1,0,0,0)	0.6348	0.6348	0.4232	0.4232	0.4232	0.0000	0.6348	0.7787	*C7>C1,C5>C6,C7>C1, C8>C6>C8>C6*
(0,0,0,0,0,1,0,0)	3.2575	3.2575	2.1717	2.1717	2.1717	3.9959	3.2575	0.0000	*C8>C6>C8*
(0,0,0,0,0,0,1,0)	0.6348	0.6348	0.4232	0.4232	0.4232	0.7787	0.6348	0.0000	*C1,C5>C6,C7>C1, C8>C6>C8>C6*
(0,0,0,0,0,0,0,1)	2.7572	2.7572	1.8381	1.8381	1.8381	3.3821	2.7572	0.0000	*C6>C8>C6*
Average	1.5930	1.5930	1.0620	1.0620	1.0620	1.8568	1.5930	0.0973	
Total	12.7442	12.7442	8.4961	8.4961	8.4961	14.8542	12.7442	0.7787	

A complete version of the fuzzy cognitive map is shown in Figure 2.

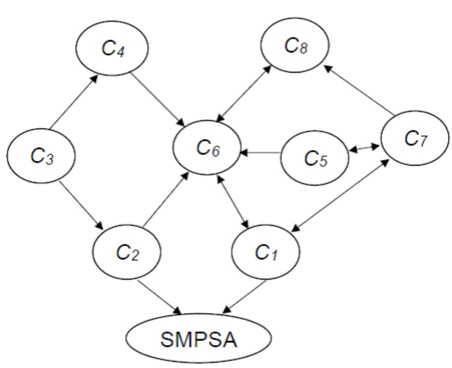

Figure 2: Induced attributes on an IRM.

The analysis of Table 5 displays the weighted sum of attributes and trigger patterns in the relationship between attributes that are the causal and trigger factors of students' mathematics problem-solving ability (SMPSA). Figure 2 illustrates the induction of the actual relationship between the attributes using a triangular fuzzy cognitive map. In the order of ranking, obtained from the results of expert consensus, the ranking of attributes according to priority is $C_6 > C_7 > C_1 > C_2 > C_5 > C_4 > C_3 > C_8$ based on their respective weights, namely executive function, metacognition, cognition, behavioural performance, attention, motivation, emotion and working memory. However, this decision and orientation cannot be confirmed until the real representation is shown by the IRM. The generated IRM will depict the actual situation. The graphical representation of the relationship will be readily visible based on the resulting IRM and the cyclic induction that is formed. The cause of the induction will be discovered through this cycle. Therefore, this IRM is very useful to determine the causal and trigger factors for any event to be studied. Among the cyclic inductions obtained is involving the outburst of emotion (C_3) which leads to two cycles. First, increasing motivation (C_4) affects the attribute executive function (C_6) and cognition (C_1) and further leads to problem-solving ability. The second is emotion affects behavioural performance (C_2), and also triggers the activation of executive function. Another example of a cycle is the one triggered by metacognition (C_7), which affects attention (C_5) and cognition (C_1) and subsequently activates executive function (C_6).

Based on the results of Table 5 and this IRM, it is clear that the executive function attribute is the main cause of SMPSA. Meanwhile, the attributes that trigger SMPSA are emotion (C_3) and metacognition (C_7). In addition, this finding shows that executive function, working memory and metacognition form a dynamic system where these elements complement each other and act actively when students solve mathematics problems. This finding is in line with and describes in detail what is meant by problem-solving theory based on cognitive and constructivist perspectives [17]. Overall, these results can explain the complexity that was previously difficult to translate, especially how to get an overview of the process and the relationship between cognitive and behavioural elements when students solve problems.

5 Conclusions

This study has successfully proven that the triangular fuzzy cognitive maps procedure can unravel issues and problems in analysing the causal and trigger factors for SMPSA. This study can also translate and explain that intelligent computational methods such as fuzzy systems are better than conventional methods such as mean analysis, correlation and so on that only report results through numerical values and graphs alone, but there are still questions that cannot be explained especially in terms of things that involve very complex variable relationships (factors). Therefore, this method is highly recommended to analyse relationships and see interaction patterns between variables. However, the constraint in carrying out this method is that it requires the wisdom of the researcher in preparing the interview text that can comply with the semi-quantitative procedure. That is, it needs to be summarised and the focus is only on determining the strength of the relationship between the variables. For this reason, to complete and ensure that this method is consistent and flexible, it needs to be expanded to various fields, not only in explaining the relationship between variables in the learning process. In addition, it is also recommended that future researchers develop this method by producing a hybrid with other appropriate methods. For example combining with the fuzzy Conjoint, Vikor, DEMATEL, AHP, ANP, TOPSIS, PROMETHEE or any methods in the group of fuzzy multi-criteria decision-making for consistency.

Acknowledgement This research was supported by Ministry of Higher Education (MOHE) through Fundamental Research Grant Scheme (FRGS/1/2022/STG06/UMT/02/4, Grant No. 59722).

Conflicts of Interest The authors declare no conflict of interest.

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